Partially Overlapping Channel Assignment Based on "Node Orthogonality" for 802.11 Wireless Networks

Yong Cui, *Member, IEEE,* Wei Li, Xiuzhen Cheng, *Member, IEEE,* Biao Chen

Abstract—In this study, we investigate the problem of partially overlapping channel assignment to improve the performance of 802.11 wireless networks. We first derive a novel interference model that takes into account both the channel separation and the physical distance separation of two nodes employing adjacent channels. This model defines "node orthogonality", which states that two nodes over adjacent channels are orthogonal if they are sufficiently physically separated. We propose an approximate algorithm **MICA** to minimize the total weighted interference for throughput maximization. To tradeoff throughput with fairness, we formulate a generalized optimization problem that maximize the bandwidth utilities and present an approximate algorithm **MUCA**. Rigorous mathematical analysis is provided to prove that the performance ratios of both **MICA** and **MUCA** are equal to 2. Extensive simulations have been performed to validate our design and to compare the performances of our algorithms with those that are the current state-of-the-art.

Index Terms—802.11 wireless networks, node orthogonality, channel separation, interference factor, partially overlapping channels, physical distance separation.

1 Introduction

The increasing popularity of Wireless Local Area Networks (WLANs) has led to a dramatic increase in the density of Access Points (APs) in many real-world applications. High node density results in strong interference and poor network performance [1]. Thus, multichannel communications has been proposed as a viable approach to mitigate such problems [2].

Nevertheless, mainstream research focuses on assigning non-overlapping channels to interfering nodes [2]–[8]. Under such a consideration, two interfering nodes can simultaneously transmit without interfering with each other only if their channels are orthogonal. However, because the number of non-overlapping channels is very limited (802.11b/g defines only 3 orthogonal channels), interference can not be completely eliminated in practical settings.

Recent studies indicate that utilizing partially overlapping channels to facilitate interference mitigation has the following advantages [9]–[13]:

- **Improve the full-range channel utilization**. When adjacent channels are considered, all channels have some chance of being utilized.
- **Improve the network throughput**. Adjacent channels can help to reduce the number of contending nodes on each channel, resulting in an increase in the network throughput.

These advantages are attributed to the fact that *partially overlapping channels do not cause interference with each other if the two nodes (i.e., the transmitter of one channel, and the receiver of the other adjacent channel) are sufficiently physically separated*. Unfortunately, this property is not fully investigated as it should be. The interference models in [9]–[11] only consider the channel separation. Though Yong et al. [12] present a weighted conflict graph by taking into account both the channel separation and the physical distance, its interference indicator is a 0-1 variable that cannot accurately reflect the degree of interference from different adjacent channels with different physical distances.

In this paper, we investigate channel allocation by considering all channels equally. We first propose a novel *Interference Factor* I_c that captures the degree of interference between two channels at different positions. This interference factor is employed to formulate an interference minimization problem for partially overlapping channel assignment to maximize the aggregated network throughput. We also generalize our problem formulation to consider bandwidth utility, which captures the tradeoff between throughput and fairness. Two approximate algorithms, MICA and MUCA, to tackle the two optimization problems via relaxation and rounding, are proposed. MICA, which stands for *Minimum Interference for Channel Allocation*, minimizes the sum of the weighted interference; MUCA, which stands for *Maximum Utility*

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for Channel Allocation, maximizes the bandwidth utility of all users. A rigorous heoretical analysis proves that both algorithms have a performance ratio of 2. The multifold contributions of the paper are summarized as follows.

- We present a novel interference model characterized by the interference factor I_c, which takes into account both the channel separation and the physical distance separation of two nodes. Compared to traditionally defined interference factors, which are usually 0-1 binary variables, I_c is a normalized real number in $[0, 1]$. Thus, it can model the interference intensity between two nodes more accurately.
- Based on our interference model, we prove that the problem of maximizing the aggregated throughput is equivalent to that of minimizing the sum of the weighted interference. According to this result, we formulate the problem **Min-Ic** for partially overlapping channel assignment to minimize the total interference, which is simpler than directly solving the problem of throughput maximization. An approximate algorithm **MICA** with a bounded performance ratio of 2 is proposed and analyzed.
- We also investigate the tradeoff between throughput and fairness. A generalized optimization problem **G-CAP** is formulated to exploit the tradeoff for different fairness criteria with different λ values. An approximate algorithm termed **MUCA** with a performance ratio of 2 for maximizing the bandwidth utility is proposed and analyzed.
- We perform comparisons based extensive simulations to verify the performance of our algorithms. The results indicate that our algorithms are superior to popular ones that also utilize partially overlapping channels for channel assignment. These results also validate the effectiveness of our interference factor I_c.

Note that the interference factor, I_c, is defined over a pair of nodes that operate on different channels. It considers both the physical distance separation between the two nodes, and their channel separation. Section 3.3 presents a detailed analysis on the properties of I_c. Traditionally, we claim that *two non-overlapping channels are orthogonal* since they do not cause interference with each other. Correspondingly, we define "node orthogonality" as *two nodes are orthogonal if they cause zero interference with each other no matter which channels they are working on*. In other words, if the $I_c = 0$ between two nodes, they are orthogonal. This notation of node orthogonality captures the fact that two nodes do not interfere with each other if they are sufficiently physically separated even though they are working on two adjacent channels. In this paper, we focus on the channel assignment based on node orthogonality.

The rest of the paper is organized as follows. We summarize the related work in Section 2. The network and interference models are described in Section 3. In Section 4, we investigate the problem of interference minimization and throughput maximization. A generalized problem that considers the tradeoff between throughput and fairness is studied in Section 5. Then, after reporting our performance evaluation in Section 6, we conclude this paper in Section 7.

2 Related Work

Although many research effort has been made for channel assignment in cellular networks, the corresponding models are not applicable to 802.11 wireless networks because of the essentially different characteristics between these two types of networks [14]. Due to the explosive growth of 802.11-based wireless networks in recent years, channel allocation has become an important and challenging issue. In particular, orthogonal channel assignment for interference elimination and throughput enhancement has been extensively studied. However, the number of orthogonal channels is quite limited in a real network, which may result in a sharp decline in the network performance under high interference scenarios [9], [11]. In contrast, it has been demonstrated that exploiting partially overlapping channels could reduce the number of contending nodes, enhance the channel re-use, and improve the network throughput [9]–[11], [13]. This paper investigates adjacent channel assignment for performance enhancement. Therefore we only outline the most related work that considers partially overlapping channel assignment in this section.

Since the interference models for orthogonal channels are not applicable to adjacent channels, a new interference model is needed. In [9]–[11], [15], a simple interference factor is defined to be the amount of the spectral overlap between two adjacent channels. This interference factor does not consider the physical distance variations of the node pair employing the adjacent channels. Ding *et al.* [12] introduce a 0-1 binary interference indicator to label whether two links interfere with each other by considering both the channel separation and the link distance. However, this interference indicator does not reflect the interference intensity, which means that it does not completely investigate both the channel and the distance separations. Wang *et al.* [16] derive a packet reception ratio based on the received signal strength (RSSI) to account for the interference caused by overlapping channels in real-time sensor networks. However, this model is not suitable for interference analysis due to the concern that RSSI is usually very sensitive to the dynamism of the environment.

In [10], Mishra *et al.* address weighted channel assignment based on graph coloring, where the weight is the total effect of interference on all users falling in the overlapping area of two APs. Two algorithms, ADJ-minmax and ADJ-sum, are proposed, with objectives of minimizing the maximum interference among all interfering APs and minimizing the sum of the weights on all conflict edges, respectively. A centralize algorithm, Randomized Compaction (RC), is proposed in [11]. The goal of this algorithm is to minimize the maximum conflict vector that consists of the total number of nodes interfering with each user arranged in non-increasing order. RC starts with a random channel assignment and refines the result iteratively. Ko *et al.* [15] use a simple greedy algorithm to allocate channels to multi-radios in wireless mesh networks. The greedy choice at each step is the channel that minimizes its local interference. Liu *et al.* [17] propose a genetic algorithm (GA) for link scheduling in wireless mesh networks by allocating partially overlapping channels. However, the performance of the algorithm depends on the number of generations and the algorithm takes a long time to converge.

3 Network and Interference Models

3.1 Network Model

In this paper, we consider an IEEE 802.11-based WLAN consisting of N APs, M users, and K channels. A user associates to the AP with the highest Received Signal Strength Indication (RSSI). An AP together with its associated users form a Basic Service Set (BSS). All nodes belonging to the same BSS operate on the same channel, i.e., the AP's channel. Since APs in close neighborhood may be assigned partially overlapping channels, BSSs might interfere with each other.

The objective of this study is to investigate adjacent channel assignment for downlink network performance maximization. Note that we choose to focus on downlink, in which data is sent by APs to users, because it produces the dominate traffic for many real-world applications such as in social networks [1], [7]. Adjacent channel assignment to maximize uplink performance will be investigated in our future research. For downlink, the experienced interference of an AP depends on the received power from other APs. There are three main factors that affect the received power from an interfering AP: (i) the transmission power of the interfering AP, (ii) the channel separation, and (iii) the physical distance between the two APs.

For simplicity, we assume that all APs transmit at the same power p. But the proposed interference model and algorithms can be easily extended to the case when the transmit powers vary.

3.2 Channel Adjacency Degree

The IEEE 802.11 standard defines a set of discrete channels for radios to operate on. The transmitter should follow the Transmit Spectrum Mask defined in the standard when allocating a power to each frequency in the channel band, with the center frequency of the channel band receiving the highest power. At the receiver side, a band-pass filter, again defined by the standard, is employed to capture the transmitted power. When the transmitter and receiver operate over the same channel, the receiver could capture the most amount of power from the transmitter; if they tune to different channels, the received power should be only a fraction. If the operating channels are totally non-overlapping, the receiver cannot capture any power from the transmitter, resulting in minimum interference at the receiver. To characterize this relationship, we introduce the concept of *channel adjacency degree*, which is defined as the amount of transmit power covered by the receiver band-pass filter:

$$\phi(c_i, c_j) = \int_{-\infty}^{+\infty} S_t(f) S_r(f-\tau) df, \tag{1}$$

where c_i and c_j are the transmit and receive channel numbers, respectively, $S_t(f)$ is the transmit power distribution across the frequency spectrum, $S_r(f)$ is the band-pass filter's frequency response, and τ is the channel separation in MHz. Note that Eq. (1) is used to define the adjacency channel *interference factor* in [11]. We decide to give Eq. (1) a different name because $\phi(c_i, c_j)$ only characterizes the channel separation. Two channels that are not sufficiently separated in the frequency domain could result in zero interference if they are used in two positions that are sufficiently separated in the space domain. In the next section, we will introduce our definition of the *interference factor* that characterizes both the channel separation and the space separation. As an example, we illustrate the channel adjacency degree of 802.11b.

In 802.11b, two immediate adjacent channels are separated by 5 MHz. The transmit power distribution and the band-pass filter's frequency response are defined as follows:

$$S_r(f) = S_t(f) = \begin{cases} -50\text{dB} & \text{if } |f - F_c| > 22\text{MHz}, \\ -30\text{dB} & \text{if } 11\text{MHz} < |f - Fc| < 22\text{MHz}, \\ 0\text{dB} & \text{otherwise}, \end{cases} \tag{2}$$

where F_c is the channel center frequency. Eq. (2) tells us that two channels c_i and c_j are orthogonal, i.e., $\phi(c_i, c_j) = 0$, if and only if the separation of their channel center frequencies is larger than 22 MHz since the bandwidth of each channel is 22 MHz. In other words, the channel separation of two orthogonal channels, in terms of channel numbers, is at least 5. Therefore, 802.11b provides a total of 3 non-overlapping channels.

3.3 Interference Model

Since $\phi(c_i, c_j)$ only considers the channel separation, it is not the right parameter to characterize interference. To introduce our interference model for partially overlapping channels, we start from a simple case that consists of AP i transmitting at channel c_i and AP j receiving at channel c_j. Let N_0 be the noise experienced by AP j, and γ_{th} be the SINR threshold for successful communications. Then, to correctly decode a signal, the following condition needs to be held:

$$\gamma_{th} < \frac{\phi(c_i, c_j) p d_{ij}^{-\alpha}}{N_0}, \tag{3}$$

where p is the transmission power, d_{ij} is the physical distance between AP i and AP j, and α is the shadowing factor ranging from 2.0 to 5.0 [18]. Note that this inequality indicates that a transmission made on a channel c_i can be correctly received at a partially overlapping channel c_j as long as the received signal is strong enough. Based on this observation, we define the *adjacent channel transmission range* between c_i and c_j, denoted by $D_t(c_i, c_j)$, as follows:

$$D_t(c_i, c_j) = \sqrt[\alpha]{\frac{p\phi(c_i, c_j)}{\gamma_{th} N_0}}. \quad (4)$$

Correspondingly, we define

$$D_i(c_i, c_j) = \beta(c_i, c_j) D_t(c_i, c_j) \phi_(c_i, c_j)^{\frac{1}{\alpha}}$$

to denote the *adjacent channel interference range* between channels c_i and c_j, where $\beta(c_i, c_j)$ is the coefficient characterizing the impact of channel separation on the interference range, and $D_t(c_i, c_j)\phi_(c_i, c_j)^{\frac{1}{\alpha}}$ represents the transmission range between two nodes in the same channel. Ding *et al.* [12] conduct an experimental study to obtain the settings of $\beta(c_i, c_j)$, summarized in Table 1, for different channel separations under different AP transmission bit rates in 802.11b networks.

TABLE 1
The value of β

channel separation	0	1	2	3	4	5
2Mb/s	2	1.125	0.75	0.375	0.125	0
5.5Mb/s	2	1	0.625	0.375	0.125	0
11Mb/s	2	1	0.5	0.375	0.125	0

Now, we are ready to define our *interference factor*, which takes both the physical distance separation and the channel separation into consideration. For two APs i and j separated by a physical distance of d_{ij}, the normalized interference factor between i and j, denoted by $I_c(i, j)$, is mapped to a real number in $[0, 1]$ and defined by Eq. (5):

$$I_c(i, j) = 1 - \frac{\min\{d_{ij}, D_i(c_i, c_j)\}}{D_i(c_i, c_j)}. \quad (5)$$

From Eq. (5), it can be observed that $I_c(i, j)$ has the following properties: (i) For a fixed physical distance d_{ij}, $I_c(i, j)$ monotonically decreases with the channel separation. (ii) For a fixed channel separation, $I_c(i, j)$ monotonically decreases with the physical distance. (iii) $I_c(i, j)$ is a real number in $[0, 1]$. A larger value indicates more serious interference. When $I_c(i, j) = 0$, AP i and j can transmit simultaneously without interfering with each other. Note that properties (i) and (ii) of $I_c(i, j)$ are consistent with the observations obtained from real-world experiments [9], [11]. In other words, we use $I_c(i, j)$ to summarize the observations in [9], [11] mathematically. The property (iii) describes the degree of interference of different node pairs. $I_c(i, j)$ is unitless, and is normalized to the range $[0, 1]$. Therefore it can be used to compare the interference degree of different node pairs, no matter which channels the transmitter and receiver are using.

Also note that $I_c(i, j)$ can be used to define "node orthogonality". Traditionally, orthogonality refers to the independency among channels where two channels are orthogonal if and only if they are non-overlapping. With $I_c(i, j)$, the definition of orthogonality can be extended to nodes: *two nodes are orthogonal if and only if their $I_c(i, j)$ is equal to 0.* Indeed, the focus of this paper is to investigate the node orthogonality for channel assignment to decrease interference and improve performance.

4 Throughput Maximization with Partially Overlapping Channels

4.1 Problem Formulation

Consider the BSS of AP j operating on channel c_j. The bit rate of a user is determined by its experienced SINR. Let γ_{ij} denote the SINR of user i associated with AP j over c_j. In the partially overlapping channel scenario, we have:

$$\gamma_{ij} = \frac{p d_{ij}^{-\alpha}}{\sum_{k=1}^{N} \phi(c_k, c_j) p d_{ik}^{-\alpha} + N_0}, \quad (6)$$

where the denominator captures the interference experienced by user i caused by APs working at channel c_k. The corresponding bit rate r_{ij} can be calculated based on Shannon's capacity theory:

$$r_{ij} = B \log_2(1 + \gamma_{ij}),$$

where B is the channel bandwidth.

Let $b_i, i = 1, 2, \cdots, M$, be the effective bandwidth of user i. The bandwidth of user i obtained from AP j is denoted by b_{ij}. Let x_{ij} be the binary association coefficient of user i and AP j with $x_{ij} = 1$ if and only if the user i is associated with AP j. Note that x_{ij} is a constant since a user selects the AP with the strongest RSSI from all its available APs. Therefore, the user bandwidth is determined by $b_i = \sum_{j=1}^{N} x_{ij} b_{ij}$. For a given user-AP association $\{x_{ij}\}$, our goal is to maximize the aggregated throughput in the network by allocating partially overlapping channels to APs. The objective function is defined as follows:

$$\sum_{i=1}^{M} b_i = \sum_{i=1}^{M} \sum_{j=1}^{N} x_{ij} b_{ij}, \quad (7)$$

where b_{ij} is calculated by the following equation:

$$b_{ij} = \begin{cases} r_{ij}, & \text{if } r_{ij} = \max_{k \in U_s(j)} \{r_{kj}\}, \\ 0, & \text{otherwise}, \end{cases} \quad (8)$$

with $U_s(j)$ being a user set consisting of all users associated with AP j. Note that Eq. (8) indicates that to obtain the maximum aggregated throughput, only the user with the highest bit rate is allowed to communicate in the multi-rate environment.

For this optimization problem we need to consider the following two constraints: (i) Each AP is allowed to access only one channel. Let $Y_{N\times K} = \{y_{jh}\}$ be a binary matrix indicating the channel assignment, where $y_{jh} = 1$ if and only if AP j transmits over channel c_h. With this notation, b_i can be expressed by $b_i(Y)$. (ii) The SINR at user i must be at least $\gamma_{th} > 0$, i.e., $\gamma_{ij} \geq \gamma_{th}$ for $j = 1, 2, \cdots, N$.

In summary, mathematically the partially overlapping channel allocation for throughput maximization can be formulated by the following optimization problem, with y_{jh} as the set of variables to be determined:

$$\max \quad \sum_{i=1}^{M} b_i(Y) \tag{9a}$$

$$\text{s.t.} \quad \sum_{h=1}^{K} y_{jh} = 1, 1 \leq j \leq N, \tag{9b}$$

$$\sum_{j=1}^{N} x_{ij}\gamma_{ij} \geq \gamma_{th}, 1 \leq i \leq M, \tag{9c}$$

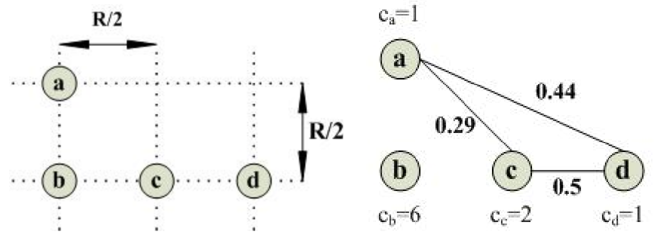
$$y_{jh} \in \{0, 1\}, 1 \leq j \leq N, 1 \leq h \leq K. \tag{9d}$$

For simplicity, the above optimization program is referred as the Original Channel Allocation Problem (**O-CAP**). Eq. (9a) is our objective function with each b_i calculated by Eq. (7). The constraint (9b) shows that an AP is allowed to access only one channel; the constraint (9c) indicates that the SINR between user i and AP j must meet the communication requirement; and the constraint (9d) specifies the range of the variable y_{jh}.

4.2 Throughput vs. Interference

In this subsection, we study the relationship between throughput and interference when partially overlapping channels are employed. Given a channel assignment, we can construct a *weighted interference graph* $G(V, E)$ such that each AP corresponds to a node in V and an edge with a weight $I_c(i, j)$ between two nodes i and j exists if and only if $i, j \in V$ and $I_c(i, j) > 0$. An example is shown in Fig. 1(a), in which four APs, a, b, c, and d, form an 802.11 wireless network with a single data rate of 11 Mbps and a unique transmit power p. Each AP is placed on a grid point, and the grid is a square with a side length of $\frac{R}{2}(R = \sqrt[4]{\frac{p}{\gamma_{th}N_0}})$. Fig. 1(b) illustrates the weighted interference graph for the channel assignment $c_a = 1$, $c_b = 6$, $c_c = 2$, and $c_d = 1$, with the weights computed by Eq. (5).

Based on the weighted interference graph, we construct the following optimization problem (Eq. (10)) to minimize the sum of the weighted interference, which is referred as **Min-Ic**. Note that **Min-Ic** is simpler compared to **O-CAP** as the minimum SINR requirements of the users are removed.



(a) A network of four APs. (b) The weighted interference graph.

Fig. 1. An example of the construction of the weighted interference graph.

$$\min \quad \sum_{j=1}^{N}\sum_{k=1}^{N} w_{kj} I_c(k, j) \tag{10a}$$

$$\text{s.t.} \quad \sum_{h=1}^{K} y_{jh} = 1, 1 \leq j \leq N, \tag{10b}$$

$$y_{jh} \in \{0, 1\}, 1 \leq j \leq N, 1 \leq h \leq K, \tag{10c}$$

where $w_{kj} = \sum_{i=1}^{M} x_{ij}(\frac{d_{ik}}{d_{jk}})^{-\alpha}$.

Theorem 1: For a given user-AP association, $\{x_{ij}\}$, **O-CAP** and **Min-Ic** are equivalent.

Proof: The objective function of **O-CAP** is $\sum_{i=1}^{M} b_i$. Since x_{ij} is a constant, maximizing the sum of b_i is equivalent to maximizing $\sum_{i=1}^{M}\sum_{j=1}^{N} r_{ij}$, where r_{ij} is the bit rate between user i and AP j. For an AP k that may interfere user i, the corresponding interference factor is denoted by $I_c(k, i)$. We plan to prove our theorem in two steps as follows.

First, from Eqs. (4), (5), and (6), γ_{ij} can be expressed as a monotone function of $I_c(k, i)$:

$$\begin{aligned} \gamma_{ij} &= \frac{pd_{ij}^{-\alpha}}{\sum\limits_{k=1}^{N} \phi(c_k, c_j) p d_{ik}^{-\alpha} + N_0} \\ &= \frac{pd_{ij}^{-\alpha} N_0^{-1} \gamma_{th}^{-1}}{\sum\limits_{k=1}^{N} \phi(c_k, c_j)^{-1}[\beta(c_k, c_i)(1 - I_c(k, i))]^{-\alpha} + 1} \end{aligned} \tag{11}$$

Since p, d_{ij}, γ_{th}, and N_0 are constants, when $I_c(k, i) \neq 1$, γ_{ij} increases as $I_c(k, i)$ decreases; correspondingly, the bit rate r_{ij} increases when $I_c(k, i)$ decreases.

Then, we can show that $I_c(k, i)$ can be replaced by $I_c(k, j)$ when $I_c(k, i) \neq 1$. Since user i associates with AP j, we have $D_i(c_k, c_j) = D_i(c_k, c_i)$. From Eq. (5), we obtain:

$$1 - I_c(k, i) = \frac{d_{ik}}{d_{jk}}(1 - I_c(k, j)). \tag{12}$$

Substituting Eq. (12) into Eq. (11) yields Eq. (13):

$$\begin{aligned}\gamma_{ij} &= \frac{pd_{ij}^{-\alpha}}{\sum_{k=1}^{N}\phi(c_k,c_j)pd_{ik}^{-\alpha}+N_0} \\ &= \frac{pd_{ij}^{-\alpha}N_0^{-1}\gamma_{th}^{-1}}{\sum_{k=1}^{N}\phi(c_k,c_j)^{-1}[(\frac{d_{ik}}{d_{jk}})\beta(c_k,c_i)(1-I_c(k,i))]^{-\alpha}+1}.\end{aligned} \quad (13)$$

Thus, maximizing $\sum_{i=1}^{M}\sum_{j=1}^{N} r_{ij}$ is equivalent to *minimizing* Eq. (14):

$$\begin{aligned}&\sum_{i=1}^{M}\sum_{j=1}^{N}(\sum_{k=1}^{N} x_{ij}(\frac{d_{ik}}{d_{jk}})^{-\alpha} I_c(k,j)) \\ =& \sum_{j=1}^{N}\sum_{k=1}^{N}(\sum_{i=1}^{M} x_{ij}(\frac{d_{ik}}{d_{jk}})^{-\alpha}) I_c(k,j) \\ =& \sum_{j=1}^{N}\sum_{k=1}^{N} w_{kj} I_c(k,j).\end{aligned} \quad (14)$$

Therefore, **O-CAP** and **Min-Ic** are equivalent for a given user-AP association $\{x_{ij}\}$. □

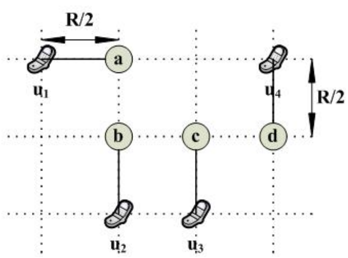


Fig. 2. An example to illustrate the relationship between the throughput maximization and the interference minimization.

TABLE 2
The solution of Min-Ic

Solution	Assignment	Throughput(b/s)	Interference
Optimal	(6,2,4,1)	24.06	0.00
Feasible 1	(6,4,1,2)	16.85	0.50
Feasible 2	(1,6,1,6)	16.67	1.15
Feasible 3	(1,1,1,1)	7.33	3.84

Theorem 1 indicates that we can solve a relatively simpler problem **Min-Ic** in stead of the more complex **O-CAP** to compute the channel assignment for throughput optimization. The example illustrated in Fig. 2 is used to validate the relationship between the aggregated throughput and the sum of the weighted interference. In this example, four APs (a, b, c, d) and four users (u_1, u_2, u_3, u_4) form a network with the solid edges indicating the user-AP association and $R = \sqrt[4]{\frac{p}{\gamma_{th}N_0}}$. Since each AP has only one user, the aggregated throughput is the sum of the bit rates of all users. There are six available partially overlapping channels, with only two non-overlapping ones, i.e., 1 and 6. All APs transmit with the same power and the same bit rate (11Mbps). The optimal channel assignment by **Min-Ic** and the corresponding total throughput and total interference are reported in the first row of Table 2. For comparison purposes, we also provide the throughput and total interference of other feasible channel assignment solutions.

4.3 An Approximate Algorithm for Min-Ic

In this section, we propose a centralized approximate algorithm termed **M**inimum **I**nterference for **C**hannel **A**llocation (**MICA**), for the problem **Min-Ic**. **MICA** is executed by placing a network manager to collect the necessary information based on which to calculate the channel assignment. It consists of three steps as shown in Alg. 1.

Algorithm 1 MICA

1: Obtain the fractional solution $\{y_{jh}^f\}$ by solving a relaxed optimization.
2: Obtain the integral solution $\{y_{jh}\}$ by a rounding process.
3: Assign the channels to APs

The basic idea is to relax the 0-1 binary variable y_{jh} such that each AP is allowed to access multiple channels, i.e., $y_{jh} \in [0, 1]$. Actually, the fractional y_{jh} implies the contribution done by the channel c_h to the total bandwidth of AP j. Under this relaxation, we compute a fractional channel allocation by resolving a relaxed optimization problem (Section 4.3.1). Then, we use a rounding process to obtain an integral solution of y_{jh} (Section 4.3.2). Based on this solution, we assign adjacent channels to all APs.

4.3.1 Relaxation of Min-Ic

The first step is to relax the binary variable y_{jh} such that $0 \le y_{jh} \le 1$. That is, an AP is allowed to access multiple channels. In such a scenario, AP j selects channel c_h if $y_{jh} > 0$. Let $I_c^f(k, c_k, j, c_j)$ be the interference between AP k in channel c_k and AP j in channel c_j. Thus, the total interference between AP k and j is:

$$I_c(k,j) = \sum_{h=1}^{K} y_{jh} \sum_{c_k=1}^{K} I_c^f(k, c_k, j, h), \quad (15)$$

where $I_c^f(k, c_k, j, h) = 1 - \frac{\min\{d_{kj}, D_i(c_k,h)\}}{D_i(c_k,h)}$.

Substituting the $I_c(k, j)$ in Eq. (10) with that in Eq. (15), we obtain the corresponding relaxed optimization problem that is referred as **RMin-Ic**:

$$\min \quad \sum_{j=1}^{N}\sum_{k=1}^{N} w_{kj} \sum_{h=1}^{K} y_{jh} \sum_{c_k=1}^{K} I_c^f(k, c_k, j, h) \tag{16a}$$

$$\text{s.t.} \quad \sum_{h=1}^{K} y_{jh} \leq 1, 1 \leq j \leq N, \tag{16b}$$

$$y_{jh} \in [0,1], 1 \leq j \leq N, 1 \leq h \leq K. \tag{16c}$$

This problem can be solved in polynomial time. The fractional optimal solution to the problem **RMin-Ic** is denoted by $I(Y^f)$.

4.3.2 Rounding for Min-Ic

In this step, we use the rounding algorithm proposed in [19] to obtain an integral assignment matrix Y. That is, replacing the fractional access coefficient $\{y_{jh}^f\}$ by a 0-1 variable $\{y_{jh}\}$ that encodes the desired assignment of channels to APs. The main idea is to construct a weighted bipartite graph based on the fractional assignment and find the minimum-weight matching for the problem **Min-Ic**. First, we introduce a definition to be used in the rounding process.

Definition 1: The experienced interference of AP j on channel c_h is defined as

$$I_{jh}(a) = \sum_{k=1}^{N}\sum_{c_k=1}^{K} w_{kj} I_c^f(k, c_k, j, h), \tag{17}$$

where $w_{kj} = \sum_{i=1}^{M} x_{ij} (\frac{d_{ik}}{d_{jk}})^{-\alpha}$.

The detail of the rounding scheme is as follows. First, we construct a bipartite graph $G_B(Y) = (A, V, E)$, where the set A represents the APs in the network, and the set V consists of the channels denoted by $V = \{v_{hs} : h = 1, \ldots, K, s = 1, \ldots, S_h\}$, with $S_h = \lceil \sum_{j=1}^{N} y_{jh}^f \rceil$. This means that each channel may have multiple nodes in V. The edges in $G_B(Y)$ are constructed in the following way. For each channel c_h, we renumber the APs according to their non-increasing experienced interference $I_{jh}(a)$. If $S_h \leq 1$, there is only one node v_{h1} corresponding to the channel c_h. For each $y_{jh}^f > 0$, add an edge $e(a_j, v_{h1})$ to E, and set $y^f(a_j, v_{h1}) = y_{jh}^f$, where $y^f(e)$ is the *access weight* of the corresponding AP and the channel. Otherwise, find the minimum index j_s such that $\sum_{j=1}^{j_s} y_{jh}^f \geq s$. For $j = j_{s-1}+1, \ldots, j_s - 1$ and $y_{jh}^f > 0$, add an edge $e(a_j, v_{hs})$ and set $y^f(a_j, v_{hs}) = y_{jh}^f$. For $j = j_s$, add the edge $e(a_j, v_{hs})$ and set $y^f(a_j, v_{hs}) = 1 - \sum_{j=j_{s-1}+1}^{j_s-1} y^f(a_j, v_{hs})$. If $\sum_{j=1}^{j_s} y_{jh}^f > s$, add the edge $e(a_j, v_{h(s+1)})$ and set $y^f(a_j, v_{h(s+1)}) = \sum_{j=1}^{j_s} y_{jh}^f - s$. Obviously, $y^f(e)$ has the following property:

$$\sum_{j=j_{s-1}+1}^{j_s} y^f(a_j, v_{hs}) \begin{cases} = 1, & s = 1, 2, \ldots, S_h - 1, \\ \leq 1, & s = S_h. \end{cases}$$

In such a case, the weight of each edge $e(a_j, v_{hs})$ in E is defined by $I_{jh}(a)$.

Second, we find a minimum-weight matching $M(Y)$ that matches each AP node to a channel node in $G_B(Y)$. For each edge $e(a_j, v_{hs})$ in $M(Y)$, schedule AP j on channel c_h and set $y_{jh} = 1$. Set other y_{js}'s to be 0. Since the fractional assignment $\{y_{jh}^f\}$ specifies a fractional matching, such a maximal matching does exist and it determines the integral association $\{y_{jh}\}$. We use $I(Y^{int})$ to denote our integral solution.

Theorem 2: Let $I(Y^*)$ be the integral optimal solution of **Min-Ic**, then $I(Y^{int}) \leq 2I(Y^*)$.

Proof: Given a fractional assignment $\{y_{jh}^f\}$, we use the rounding scheme proposed by Shmoys and Tardos to derive the final integral assignment $\{y_{jh}^{int}\}$ in polynomial time [19], [20]. These two solutions satisfy the property that $I(Y^{int}) \leq I(Y^f) + I_{max}$, where I_{max} is the sum of the maximum experienced interferences by all APs on each channel. Obviously, $I_{max} \leq I(Y^f)$ and $I(Y^f) \leq I(Y^*)$. Thus we have $I(Y^{int}) \leq I(Y^f) + I(Y^f) = 2I(Y^f) \leq 2I(Y^*)$. That is, our solution is at most twice of the optimal in the worst case. □

We use an example to illuminate the rounding process. There are four APs and two channels. The fractional channel assignment sets $y_{11}^f = \frac{1}{3}$, $y_{12}^f = \frac{2}{3}$, and $y_{21}^f = y_{31}^f = y_{42}^f = 1$; the corresponding weights are $I_{1j}(a) = \frac{1}{2}, (j = 1, 2)$ and $I_{ij}(a) = 1, (i = 2, 3, 4)$. The bipartite graph $G_B(Y)$ is given in Fig. (3), where $y^f(a_j, v_{hs}) = \frac{1}{3}$ for dashed edges and $y^f(a_j, v_{hs}) = \frac{2}{3}$ for solid edges. Then, the minimum weight matching yields $y_{12} = y_{42} = 1$, $y_{21} = y_{31} = 1$.

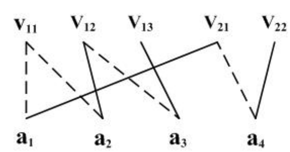


Fig. 3. An example to illustrate the rounding process.

5 Extension to Fairness

Note that in multi-rate wireless networks, maximizing the total aggregated throughput results in starvation of low rate users. Therefore fairness is usually considered as important as throughput. In this section, we extend our investigation to various trade-offs between throughput and fairness when considering partially overlapping channel allocation. For this purpose the problem **O-CAP** is expanded to yield the following Generalized Channel Allocation Problem (**G-CAP**):

$$\max \quad U(Y) = \sum_{i=1}^{M} u_i(Y) \tag{18a}$$

$$\text{s.t.} \quad \sum_{h=1}^{K} y_{jh} = 1, 1 \leq j \leq N, \tag{18b}$$

$$y_{jh} \in \{0,1\}, 1 \leq j \leq N, 1 \leq h \leq K, \tag{18c}$$

where $u_i(Y)$ is the bandwidth utility of the users and is defined according to [21]:

$$u_i(Y) = \begin{cases} \frac{b_i^{1-\lambda}}{1-\lambda}, & \text{if } \sum_{j=1}^{N} x_{ij}\gamma_{ij} \geq \gamma_{th}, \\ -\infty, & \text{otherwise}, \end{cases} \tag{19}$$

where $\lambda \geq 0$ and $\lambda \neq 1$. The utility function is convex and increasing, which is valid for many real-world applications. By appropriately selecting the value of λ, the utility function can provide various trade-offs between the network throughput and fairness. For example, the optimal problem **G-CAP** corresponds to *maximizing the aggregated throughput* when $\lambda = 0$; *proportional fairness* when $\lambda \to 1$; and *max-min fairness* when $\lambda \to +\infty$ [22]. The corresponding expressions of user bandwidth in different cases are defined as follows.

- *Proportional fairness*: also referred as *time-based fairness* when all users have the same weight [23].

$$b_{ij} = \frac{r_{ij}}{|U_s(j)|}.$$

- *Max-min fairness*: also referred as *throughput-based fairness* when all users have the same weight [24].

$$b_{ij} = \frac{1}{\sum_{k \in U_s(j)} 1/r_{kj}}.$$

For the case of maximizing the network throughput, users with lower bit rates may be starving. To avoid this, we change the utility function in Eq. (9a) to Eq. (19) so that the solution is infeasible if there exists a user that is not allocated enough bandwidth. That is, the optimal value of **G-CAP** being equal to $-\infty$ implies that there is no feasible solution to **O-CAP**. This change can guarantee successful communications for each user.

5.1 Approximate Algorithm for G-CAP

In this section, we propose a centralized approximate algorithm termed Maximum Utility for Channel Allocation (**MUCA**) for **G-CAP**. The basic idea of **MUCA** is similar to that of **MICA**, and **MUCA** contains three steps shown in Alg. 2.

Algorithm 2 MUCA

1: Obtain the fractional solution $\{y_{jh}^f\}$ by solving a relaxed optimization.
2: Obtain the integral solution $\{y_{jh}\}$ by a rounding process.
3: Assign the channels to APs.

5.1.1 The Relaxation of G-CAP

We allow each AP to access multiple channels in the relaxation by setting $y_{jh} \in [0, 1]$. In the multi-channel scenario, we denote the SINR for user i, AP j, and channel c_h by γ_{ijh}, which is defined as

$$\gamma_{ijh} = \frac{p d_{ij}^{-\alpha}}{\sum_{k=1}^{N} \sum_{c_k=1}^{K} \phi(c_k, h) p d_{ik}^{-\alpha} + N_0}.$$

Correspondingly, let r_{ijh} be the bit rate for user i, AP j, and channel c_h, which is calculated according to

$$r_{ijh} = B \log_2(1 + \gamma_{ijh}). \tag{20}$$

Therefore, the user bandwidth can be calculated by Eq. (21):

$$b_i = \sum_{j=1}^{N} x_{ij} b_{ij} = \sum_{j=1}^{N} \sum_{h=1}^{K} x_{ij} y_{jh} b_{ijh},$$

where b_{ijh} is obtained from the utility functions defined in Section 4.1.

After relaxation, we obtain the following Relaxed Channel Allocation Problem (**R-CAP**):

$$\max \quad U(Y^f) = \sum_{i=1}^{M} (u_i(Y^f)) \tag{21a}$$

$$\text{s.t.} \quad \sum_{h=1}^{K} y_{jh}^f \leq 1, 1 \leq j \leq N, \tag{21b}$$

$$y_{jh}^f \in [0, 1], 1 \leq j \leq N, 1 \leq h \leq K. \tag{21c}$$

The constraint (21b) indicates that the total fractional channel access of AP j can not surpass 1; the constraint (21c) defines the range of the variable y_{jh}^f. Obviously, the optimal solution y_{jh}^f for **R-CAP** can be found in polynomial time. Let $U(Y^f)$ be the optimal fractional solution of **R-CAP**.

5.1.2 Rounding for G-CAP

To proceed, we first define the *AP* utility on each channel c_h.

Definition 2: The utility of AP j on channel c_h is defined as the sum of the utility of users associated with it in channel c_h, i.e.,

$$u_{jh}(a) = \sum_{i=1}^{M} x_{ij} u_{ijh} = \sum_{i=1}^{M} x_{ij} \frac{b_{ijh}^{1-\lambda}}{1-\lambda}. \tag{22}$$

The rounding process for **G-CAP** is similar to that of **Min-Ic** described in Section 4.3.2, except for the following changes:

- For each channel c_h, instead of sorting the APs in non-increasing order of the experienced interference as in **Min-Ic**, the APs are sorted in their non-decreasing order of utilities in **G-CAP**.
- In stead of associating a weight with each edge $e(a_j, v_{hs})$ in E as in **Min-Ic**, we associate a profit that is defined to be the utility of the AP in **G-CAP** with each edge.
- In stead of finding a minimum-weight matching of the constructed bipartite graph as in **Min-Ic**, we look for a maximum-profit matching $M(Y)$ for **G-CAP**.

Let $U(Y^{int})$ and $U(Y^*)$ denote the integral solution of **MUCA** and the optimal solution of **G-CAP**, respectively. The following theorem summarizes the performance of **MUCA** for the problem **G-CAP**.

Theorem 3: $U(Y^{int}) \geq \frac{1}{2}U(Y^*)$.

Proof: The IEEEproof procedure is similar to that of Theorem 2. According to [19], the rounding result has the following property: $U(Y^{int}) + U_{min} \geq U(Y^f)$, where U_{min} is the sum of the minimum AP utility of each channel. Since $U(Y^{int}) \geq U_{min}$ and $U(Y^f) \geq U(Y^*)$, we have $U(Y^{int}) \geq \frac{1}{2}[U(Y^{int}) + U_{min}] \geq \frac{1}{2}U(Y^f) \geq \frac{1}{2}U(Y^*)$. □

In other words, the approximate solution obtained from **MUCA** is not less than half of the optimal solution of **G-CAP** in the worst case. Meanwhile, our algorithm **MUCA** can guarantee the quality of user bandwidth. Note that Theorem 2 and 3 imply that our algorithms have a performance ratio of 2 when partially overlapping channels are exploited in the network.

6 Simulation

6.1 Methodology

In this section, we evaluate the performance of our proposed algorithms through an extensive simulation study, and compare their performances with those of other algorithms, i.e., Randomized Compaction (RC) [11], ADJ-sum, and ADJ-minmax [10]. Note that all of these algorithms are centralized. The RC algorithm minimizes the maximum number of interfered users, while ADJ-sum and ADJ-minmax minimize the sum of the weighted interference and the maximum weighted interference, respectively. Note that the interference factor utilized by ADJ-sum and ADJ-minmax is the adjacency channel degree defined in this paper. For **MUCA**, we consider different bandwidth utility functions by adjusting λ: maximizing the aggregated throughput with $\lambda = 0$, achieving proportional fairness with $\lambda \to 1$, and obtaining max-min fairness with $\lambda \to +\infty$.

All of these algorithms are examined according to the following performance metrics:

- The per-user throughput, and the corresponding statistics.
- The Jain's fairness index, i.e., $J = \sum_{i=1}^{M} b_i^2 / (\sum_{i=1}^{M} b_i)^2$, where a larger value of $J \in [0, 1]$ indicates a better fairness in resource management [25].
- The channel utilization ratio, which characterizes the utilization of each available channel and is defined to be the ratio of the number of APs over the channel and the total number of APs.
- The packet-drop ratio, which indicates the seriousness of collision due to adjacency or co-channel interference.

There are two different scenarios in our simulation, the uniform and the hotspot cases, with both containing 20 APs and 50-250 users in a three-dimension physical region. For the uniform case, the locations of APs and users are chosen uniformly at random in a $1400m \times 900m \times 10m$ region. In the hotspot case, to simulate high interference in the wireless environment, we place users in a certain area with $500m \times 500m \times 10m$, while APs distribute uniformly at random in a $1000m \times 500m \times 10m$ region. In the simulation, we intend to evaluate the performance of downlink, that is, the data is set by an AP to its users. The numbers of the data packets that are sent from each AP to its users are the same.

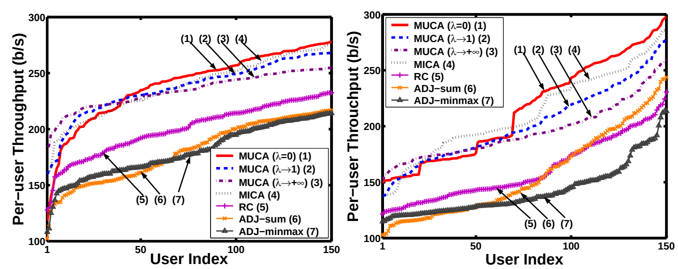
TABLE 3
Simulation Parameters

Parameter	Value
Radio Type	802.11b Radio
Number of Channels	11
Shadowing Factor (α)	4
Packet Size	512 bits
Packet Reception Model	PHY802.11b
Transmission Power	15dBm
Transmission Rate	6Mb/s

We solve both relaxed non-linear programs (Eqs. (16) and (21)) by LINGO and use QualNet as the simulator with the corresponding parameters shown in Table 3.

6.2 Numerical Results

In this subsection, we report our numerical simulation results, which are averaged over 50 runs with each taking 300s simulation time. Since the results are essentially similar, we only report the case with 20 APs and 150 users.



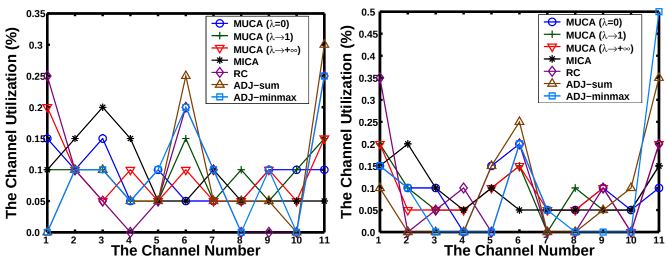
(a) the uniform case (b) the hotspot case

Fig. 4. The per-user throughput.

First, the per-user throughput is reported in Fig. 4, with the users sorted by their throughput in increasing order. We also list the per-user throughput statistics and fairness in Table 4. From Fig. 4 and Table 4, we observe that our algorithms outperform the other three algorithms in terms of per-user throughput and Jain's fairness index for both network settings. Although **MUCA($\lambda \to +\infty$)**, RC, and ADJ-minmax exploit max-min fairness in their partially overlapping channel allocation, **MUCA($\lambda \to +\infty$)** achieves the highest per-user average throughput and the largest fairness index. The superiority of our algorithms is attributed to the interference factor I_c, which helps to obtain a more appropriate channel assignment when considering the tradeoff between throughput and fairness. Note that problems **O-CAP** and **Min-Ic** are equivalent as indicated by our Theorem 1. But the numerical results are obtained

TABLE 4
The Statistics of the User Bandwidth

Scenario	Alg.	Ave.(b/s)	Std.(b/s)	Fairness Index
Uniform	**MUCA**($\lambda = 0$)	379.28	64.70	0.97
	MUCA($\lambda = 1$)	374.22	48.79	0.98
	MUCA($\lambda \to \infty$)	368.58	30.95	0.99
	MICA	375.32	52.60	0.98
	RC	300.18	46.30	0.97
	ADJ-sum	259.75	54.46	0.96
	ADJ-minmax	258.91	46.73	0.97
Hotspot	**MUCA**($\lambda = 0$)	329.56	89.63	0.93
	MUCA($\lambda = 1$)	304.49	63.57	0.96
	MUCA($\lambda \to \infty$)	290.92	49.77	0.98
	MICA	323.41	75.80	0.95
	RC	221.23	56.00	0.93
	ADJ-sum	213.04	76.84	0.88
	ADJ-minmax	184.00	56.08	0.84

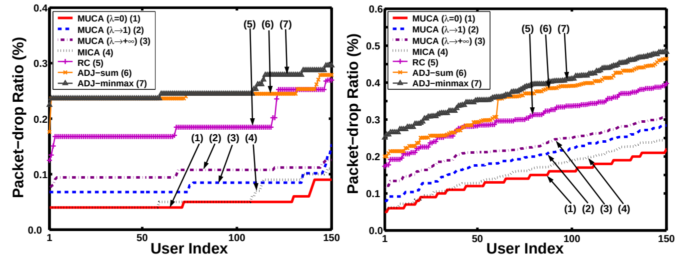


(a) the uniform case (b) the hotspot case

Fig. 5. The channel utilization.

from the two approximate algorithms **MUCA(**$\lambda = 0$**)** and **MICA**, respectively. Therefore there is a little difference between the results of **MUCA(**$\lambda = 0$**)** and **MICA**. In summary, we conclude that our algorithms are superior in terms of throughput and fairness compared to others when partially overlapping channels are exploited.

Next, we show the utilization of each channel in Fig. 5. It can be seen that in the uniform case, **MICA** and **MUCA** utilize all channels while RC, ADJ-sum and ADJ-minmax only use a part of them. Furthermore, the variances of the channel utilization produced by our algorithms are smaller than those from others. The essential reason for these two results is that our algorithms allocate channels based on "node orthogonality" which takes into account both the channel separation and the physical distance separation while others assign channels based on the traditional channel orthogonality by only considering the channel separation. That indicates that the "node orthogonality" can help to mitigate interference. In the hotspot case, **MICA** employs all available channels, which implies that our interference factor could improve the channel utilization. Nevertheless, a larger channel separation is needed to mitigate the stronger interference due to the higher user density. This indicates that the channel utilization in the hotspot case is relatively lower.



(a) the uniform case (b) the hotspot case

Fig. 6. The packet-drop ratio.

Finally, we present our results on packet-drop ratio in both network scenarios, with the users sorted by their packet-drop ratios in increasing order. Fig. 6 clearly indicates that users receive more data packets based on our algorithms. The reason is attributed to our interference factor I_c, which considers both the channel separation and the physical distance separation, generating a lower interference between two APs. In contrast, RC, ADJ-sum, and ADJ-minmax calculate the channel allocation by taking only the channel separation into account, resulting in a comparatively inefficient channel utilization and higher interference.

7 Conclusion

The widespread use of WLAN applications drives a high density deployment of APs, leading to an increase in interference and a decrease in the network performance. In this paper, we study how to mitigate interference and improve the network performance by allocating partially overlapping channels. We first establish a novel interference model characterized by the interference factor I_c, which takes into account both the channel separation and the physical distance separation of the two nodes. Based on this model, we formulate an interference minimization problem and propose a heuristic algorithm **MICA**. To investigate the tradeoff between throughput and fairness, we obtain a generalized problem to maximize the total bandwidth utility, and propose an approximate algorithm **MUCA**. We provide rigorous theoretical performance analyses and conclude that both **MICA** and **MUCA** achieve a bounded performance ratio of 2. Finally, simulation study validates the effectiveness of our design.

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